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# Bilateral Delegation in Duopoly Wage and Employment Bargaining

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## Abstract

We study bilateral delegation in wage and employment bargaining between firms and unions in a Cournot duopoly. Incentive delegation creates frictions for each party between its objectives of within-firm rent extraction and market/job stealing from the rival firm. The net effect is restraint in production resulting in a larger bargaining pie. But each player's payoff will be inversely related to his bargaining power. We also show that if players are given a choice to delegate, they will not resort to delegation when their bargaining power is sufficiently high. This is in contrast to the scenarios commonly assumed in many models.

**JEL Classifications:** J50, L13, D43.

**Key Words:** Managerial incentives, efficient bargaining, bilateral delegation, implicit collusion, job stealing

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# 1 INTRODUCTION

It is well known that firms may delegate their output decisions to managers inducing them to maximize sales rather than profit in order to steal business from their rivals (Vickers, 1985; Fershtman and Judd, 1987; Sklivas, 1987). The idea of advantage-seeking delegation, and particularly the strategic overproduction aspect of it, recur through many themes of the industrial organization literature (see, for instance, Zhang and Zhang (1997), Lambertini and Trombetta (2002), Mujumdar and Pal (2007) and Mukherjee and Tsai (2014)), and also feature in other areas like development economics (Basu *et al.*, 1997) and strategic trade theory (Das, 1997). However, when the strategic delegation model has been extended to wage bargaining (Szymski, 1994) it is seen that managers may not always maximize sales, because their incentive to steal business from the rival firm comes into a conflict with the union's wage demand.

But delegation has generally been modeled as a unilateral act, mainly occurring on the firm side, and therefore the inherent conflicts in managerial incentives have not been studied in full scale. Even in standard wage bargaining models, barring a few exceptions such as Jones (1989a), unions don't delegate the task of negotiation to the union leader.<sup>1</sup> In reality negotiation between the firm and its workers is always mediated by a union leader and a manager, and the manager also doubles as the decision maker for the firm's output market activities. Chatterjee and Saha (2013) allow bilateral delegation in wage bargaining; but their model is restricted to monopoly and hence, no strategic considerations could be accommodated.<sup>2</sup>

This paper permits delegation on both sides – the firm and the union – and studies its implication for duopoly competition as well as rent allocation within the framework of efficient wage bargaining. While both sides have a shared interest in stealing business and jobs from the rival firm, they are at conflict within the firm. We aim to study how delegation strikes a balance here. Furthermore, while the existing models assume exogenous delegation, this paper suggests one way of making the delegation decision endogenous. Therefore, not only do we study conflicts in incentives on either side of the bargaining table, but also examine if the players wish to avoid this *ex post* incentive conflict by simply not delegating in the first place.

Our main result is that bilateral delegation diffuses 'strategic aggression' and avoids overproduction, which in turn enlarges the firm surplus, and both firms move toward a cooperative outcome without any overt or tacit collusion. Unions also benefit by gaining a larger share of the organizational rent. But this mutually beneficial outcome may be attained at a cost – the bargaining power of each side exerting a negative effect on its payoff. That is to say, delegation fully undermines the institutionally determined bargaining powers by overturning the rules of rent sharing – allowing the weaker party to get a larger share of the pie, through aggressively incentivizing its delegate.

The explanation is as follows. In delegating the task of bargaining the shareholders face

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<sup>1</sup>Jones's (1989a) main objective is to show that delegation to a union leader is tantamount to making commitment.

<sup>2</sup>Jones (1989b) also consider bilateral delegation in a pure bargaining environment; but his concerns are different.

two conflicting incentives. On the one hand, they would like to steal market and hence need to encourage their manager to expand production by ‘undervaluing’ the marginal cost (which depends on the reservation wage). But on the other hand, they also want to increase their share of the organizational rent and hence expect the manager to reduce the wage bill, which in turn requires him to ‘overvalue’ the reservation wage and curb production. The first motive is the strategic motive, and it calls for a sales-oriented incentive scheme. The second motive is the bargaining motive, which calls for a profit-oriented incentive scheme. That which scheme will be chosen and which motive will dominate depends on how powerful the shareholders are vis-a-vis the workers. Greater the bargaining power of the shareholders, stronger their strategic motive. The reason is that a stronger firm can appropriate a greater share of the organizational surplus, and hence its returns to strategic actions are greater. By the same logic, smaller the bargaining power, weaker the strategic motive and stronger the bargaining motive.

The workers will also face two conflicting incentives. They share a similar strategic motive in stealing jobs from the rival firm. This requires inducing the union leader to ‘undervalue’ the reservation wage and maximize the gross wage bill. Yet at the same time they would like to press for a higher wage, which can be achieved if the union leader ‘overvalues’ the reservation wage and is geared towards net wage bill maximization. The job-stealing motive dominates when the workers have high bargaining power.

Since the bargaining power of the two parties are inversely related and the relative returns to the two objectives are different to them. Within a given firm, the weaker party’s bargaining motive will counteract the strategic motive of the stronger party. The weaker party would try to reduce production, while the stronger party would like to expand production. In effect, the strategic aggression is diffused, production of each firm is restricted and their surplus increases from the standard duopoly level. When both sides are nearly equally powerful, two conflicting incentives are in balance, and production is reduced to the collusive level. Organizational surplus in each firm reaches its maximum.

This contradicts the over-expansion result of the strategic delegation models like Fershtman and Judd (1987). The reason is that with only firm-side delegation and no within firm bargaining, the firm’s strategic motive goes overboard and in equilibrium both firms excessively produce and lose profit? In the presence of bilateral delegation, a weaker player controls the strategic aggression of its bargaining rival.<sup>3</sup> Thus, by appointing a delegate and endowing him with a different objective function, the weaker party can threaten to punish strategic overproduction and thus restrain the market stealing or job stealing motive of its stronger rival.

These results are established with general demand and cost functions in a duopoly setup. With the help of an example involving linear demand and constant returns to scale technology we also show that the distribution of the organizational pie does not align with the distribution of the bargaining power. In fact, the payoff of a given party will be inversely related to its

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<sup>3</sup>Fershtman and Judd (1987) observed that if incentive delegation occurs in a price competition model then firms move toward collusion. But in a model of quantity competition, as Szymanski (1994) has shown, the presence of a third party, like a labor union would discipline the firm. However, he assumes ‘right-to-manage’ protocol and no delegation on the union side.

bargaining power. By appointing a delegate and incentivizing him for hard bargaining a weaker party can extract greater concession from its opponent, and thus the payoff-power relationship is reversed.

In Chatterjee and Saha (2013) as well, which also deals with bilateral delegation, but only under monopoly, the conventional payoff-power relationship breaks down. In the absence of any strategic objectives, the sole purpose of delegation is to fare well in bargaining. So a weaker party counters its opponent by providing stronger incentives to its delegate, and succeeds in increasing its share of the organizational pie. However, the pie itself changes with incentives. In monopoly, both sides will always incentivize their delegates to ‘overvalue’ their own costs; the firm resorts to profit oriented delegation, and the union resorts to net wage bill oriented delegation. This leads to substantial production losses from the no-delegation monopoly level.

In contrast, in the duopoly model, as the strategic motive also comes into play the extent of bargaining motivated incentives gets moderated. So we have a range of possibilities starting from the firm trying to maximize sales, while the union trying to maximize the net wage bill, to the other extreme where the firm is trying to maximize profit and the union is trying to maximize the gross wage bill. Moreover, with an example we show that the organizational pie will always be *greater* than the duopoly no-delegation pie. Thus, it seems that bilateral delegation is potentially beneficial to the firm and the union jointly.

Finally, we extend the game to see if the players were given a choice, whether they would really choose to delegate. While a fully non-cooperative extension of the game is fairly complex and beyond the scope of the paper, we propose a simple semi-collusive variant of it to restrict our attention to the symmetric delegatin decisions (which we explain in Section 3.2). It turns out that delegation is a dominant strategy for *both* sides only if their bargaining powers are not highly uneven. But if they are, the stronger party will choose not to delegate giving rise to an asymmetric delegation scenario with only the weaker party resorting to (bargaining) delegation. This gives rise a different scenario to the ones, commonly appearing in the literature.

There is some empirical evidence of profit orientation in managerial incentive schemes. In a US medical industry report Kasinec (2006) writes that managers are discouraged to hire more staff or pay more wages by linking their bonuses mainly to companies’ annual profit and annual productivity growth targets. There is also an established literature that studies CEO and managerial compensation in the presence of unions. Singh and Agarwal (2002) showed that in the Canadian manufacturing sector union presence is associated with greater CEO pay. On the issue of union facilitating implicit collusion one finds some indirect evidence from Haskel and Martin (1992), who observed in the context of Britain that union membership positively affects the price-cost mark-up via industry concentration.

The paper is organized as follows. Section 2 presents the model and the main results for general demand and cost conditions. Section 3 studies linear demand and constant returns technology to present some simulation results, and also models endogenous delegation. Section 4 concludes.

## 2 THE MODEL

There are two identical firms, indexed 1 and 2. Labor, for simplicity, is the only input needed for production which is given by the technology  $q = q(l)$ . Assuming a concave production function  $q = q(l)$ , we write the sales revenue of the  $i$ -th firm as  $s_i = s_i(l_1, l_2)$ , which is strictly concave with the following restrictions:

**Assumption 1**  $\frac{\partial s_i}{\partial l_j} < 0$ ,  $\frac{\partial^2 s_i}{\partial l_i^2} < 0$ ,  $\frac{\partial^2 s_i}{\partial l_i \partial l_j} < 0$  for  $i \neq j$ .

The first restriction signifies that the two goods are substitutes – perfect or imperfect; the second restriction relates to concavity and the third restriction ensures that employments are strategic substitutes.

The shareholders of firm  $i$  hire a manager and offer an incentive scheme  $z_i$ , which is, as in Fershtman and Judd (1987) (in short FJ), a linear combination of sales  $s_i$  and profit  $\pi_i$  as follows.

$$z_i = \beta_i \pi_i + (1 - \beta_i) s_i(\cdot) = s_i(\cdot) - \beta_i w_i l_i. \quad (1)$$

Non-trivial delegation arises if  $\beta_i \neq 1$ , and there are two types of delegation that can arise – sales oriented delegation (i.e.  $\beta_i < 1$ ), and profit oriented delegation (i.e.  $\beta_i > 1$ ). Shareholders maximize profit  $\pi_i = s_i(\cdot) - w_i l_i$ .

The workers' union in firm  $i$  consists of  $N_i$  identical workers whose reservation wage is  $\theta$  and the objective function is  $u_i = (w_i - \theta) l_i$ . At the worker selection stage,  $l_i$  members are randomly hired and the remaining  $(N_i - l_i)$  members receive the reservation wage from outside. Workers in each firm appoint a union leader who is asked to maximize:

$$v_i = \gamma_i u_i(\cdot) + (1 - \gamma_i) w_i l_i = w_i l_i - \gamma_i \theta l_i. \quad (2)$$

Delegation is captured by  $\gamma_i \neq 1$ . If  $\gamma_i > 1$  the union leader is oriented to net wage bill maximization as opposed to  $\gamma_i < 1$  when he is oriented to gross wage bill maximization.<sup>4</sup> Effectively, the leader is induced to overvalue ( $\gamma_i > 1$ ) or undervalue ( $\gamma_i < 1$ ) the opportunity cost of the union.

Each firm's wage and employment are an outcome of Nash bargaining between the firm manager and the union leader. In both firms, the bargaining power of the union leader (and also the union) is exogenously given by  $\alpha$ ,  $0 \leq \alpha \leq 1$ , and the manager's (and also the shareholders') bargaining power is  $(1 - \alpha)$ . The reservation payoffs of all parties are zero.

In stage 1 of the game the shareholders choose  $\beta_i$  and simultaneously the unions choose respective  $\gamma_i$ . In stage 2 wage and employment (and consequently output) are determined.

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<sup>4</sup>It is conceivable that the union leader might be encouraged to negotiate for higher employment. For example, the leader might be assigned an objective function  $v_i = (w_i - \theta) l_i + \gamma_i l_i = w_i l_i - (\theta - \gamma) l_i$ . The analysis will be qualitatively similar.

## 2.1 Optimal Incentives

We start from stage 2 of the game. Given  $(\beta_1, \gamma_1), (\beta_2, \gamma_2)$  the manager and the union leader in each firm simultaneously bargain over their firm-specific wage and employment. The Nash bargaining problem is to maximize  $B_i = v_i(w_i, l_i)^\alpha z_i(w_i, l_i)^{(1-\alpha)}$  with respect to  $(w_i, l_i)$ .

The solution is given by the following two equations.

$$\frac{\partial s_i(l_i, l_j)}{\partial l_i} = \beta_i \gamma_i \theta, \quad i = 1, 2 \quad (3)$$

$$w_i = (1 - \alpha) \gamma_i \theta + \alpha \frac{s_i}{\beta_i l_i}, \quad i = 1, 2. \quad (4)$$

Solving (3) for firm 1 and 2 simultaneously we obtain the employment functions  $l_i = l_i(\beta_1 \gamma_1, \beta_2 \gamma_2), i = 1, 2$  and substituting them in (4) we obtain  $w_i = w_i(l_1, l_2)$ . Further, it is easy to check that  $\frac{\partial l_i}{\partial \beta_i} < 0$ ,  $\frac{\partial l_i}{\partial \gamma_i} < 0$  and  $\frac{\partial l_i}{\partial \beta_j} > 0$ ,  $\frac{\partial l_i}{\partial \gamma_j} > 0$ ,  $i \neq j$ .

After substituting (4) and rearranging terms we can write profit and union utility respectively in the following way:

$$\pi_i = s_i \left( 1 - \frac{\alpha}{\beta_i} \right) - (1 - \alpha) \theta \gamma_i l_i = (s_i - \theta l_i) - \alpha \left( \frac{s_i}{\beta_i} - \theta \gamma_i l_i \right) - (\gamma_i - 1) \theta l_i \quad (5)$$

$$u_i = \left( s_i \frac{\alpha}{\beta_i} + (1 - \alpha) \theta \gamma_i l_i \right) - \theta l_i = \alpha \left( \frac{s_i}{\beta_i} - \theta \gamma_i l_i \right) + (\gamma_i - 1) \theta l_i \quad (6)$$

Eq. (5) shows how profit is obtained after subtracting the wage bill from the bargaining pie  $[s_i - \theta l_i]$ . If there were no delegation, we would have had  $\pi_i = (1 - \alpha)[s_i - \theta l_i]$  and  $u_i = \alpha[s_i - \theta l_i]$ . But with delegation the payoffs change, about which we make the following observations.

1. For firm  $i$ 's profit to be positive, optimal  $\beta_i$  must exceed  $\alpha$ . To be more precise, the following inequality must hold

$$\beta_i > \frac{\alpha}{1 - (1 - \alpha) \theta \gamma_i \frac{l_i}{s_i}}.$$

2. If bargaining was the sole objective of delegation, the shareholders would set  $\beta_i > 1$  (i.e. profit orientation, rather than sales orientation) and the union would set  $\gamma_i > 1$  (i.e. net wage bill orientation). But in the presence of a strategic motive, this would imply giving away a share of the market to the rival firm, as we shall shortly see.
3. Delegation can guarantee a strictly positive payoff for either side even when a given side has the least bargaining power:  $\pi_i(\alpha = 1) > 0$  if  $\beta_i > 1$  and  $u_i(\alpha = 0) > 0$  if  $\gamma_i > 1$ . Thus, delegation acts as a substitute for low bargaining power.

Now let us derive the optimal incentives by considering the stage 1 problem. Given  $(\gamma_i, \beta_j, \gamma_j)$  shareholders in firm  $i$  maximize Eq. (5). The first order condition for profit maximization is

(after substituting (3)) for  $i = 1, 2$ , and  $i \neq j$

$$\frac{\partial \pi_i}{\partial \beta_i} = \frac{1}{\beta_i^2} \left[ \underbrace{\left\{ \beta_i^2 (\beta_i - 1) \gamma_i \theta \right\} \frac{\partial l_i}{\partial \beta_i} + \alpha s_i}_{\text{Bargaining effect } (> 0)} + \underbrace{\left( 1 - \frac{\alpha}{\beta_i} \right) \frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial \beta_i}}_{\text{Strategic effect } (< 0)} = 0, \quad (7)$$

The first term of Eq. (7) captures the bargaining effect of delegation. The second term is the duopoly induced strategic effect. The negative strategic effect encourages the shareholders to lower  $\beta$  while the positive bargaining effect encourages to raise it.

Similarly, by maximizing the union's utility as given in (6) for any given  $(\beta_i, \beta_j, \gamma_j)$ , we derive the first order condition as

$$\frac{\partial u_i}{\partial \gamma_i} = \theta \left[ \underbrace{(\gamma_i - 1) \frac{\partial l_i}{\partial \gamma_i} + (1 - \alpha) l_i}_{\text{Bargaining effect } (> 0)} + \underbrace{\frac{\alpha}{\beta_i} \frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial \gamma_i}}_{\text{Strategic effect } (< 0)} = 0. \quad (8)$$

The strategic effect is negative also for the union; but it is also directly related to their bargaining power. If their bargaining power weakens, the strategic effect of their delegation will also weaken.

**Unilateral delegation by shareholders.** Suppose the unions do not delegate ( $\gamma_i = \gamma_j = 1$ ); only the shareholders do. This case is of particular interest for the strategic delegation result popularized by Vickers (1985), Fershtman and Judd (1987), and Sklivas (1987). Not only do we generalize the strategic delegation models to wage bargaining, but also present an efficient bargaining version of Szymanski (1994).

Set  $\gamma_i = 1$  in Eq. (7) and suppose  $\beta_1^* = \beta_2^* = \beta^*$  satisfy Eq. (7) for  $i = 1, 2$ .<sup>5</sup> The following can be said about  $\beta^*$  unambiguously.

**Proposition 1** ( *Unilateral delegation by shareholders* ) Denote  $\beta^*(\alpha = 0) = \beta^S$  and  $\beta^*(\alpha = 1) = \beta^B$ . Then  $\beta^S < 1 < \beta^B$ , and  $\beta^{*'}(\alpha) > 0$  for all  $\alpha \in [0, 1]$ . Hence, there exists a unique critical value of  $\alpha$ , say  $\hat{\alpha} \in (0, 1)$ , such that at all  $\alpha \in (\hat{\alpha}, 1]$  delegation is profit oriented ( $\beta^* > 1$ ), at all  $\alpha \in [0, \hat{\alpha})$  delegation is sales-oriented ( $\alpha < \beta^* < 1$ ), and at  $\alpha = \hat{\alpha}$  there is effectively no delegation ( $\beta^* = 1$ ).

Proposition 1 shows that sales-orientation, which commonly features in strategic delegation models, should not be taken for granted. Sales-orientation occurs only when unions have no or little bargaining power<sup>6</sup>. Particularly at  $\alpha = 0$  the FJ model is replicated. With stronger

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<sup>5</sup>For this solution to be valid we assume that the Cournot stability condition holds along with the second order condition for each firm's profit maximization.

<sup>6</sup>Szymanski (1994) has shown a similar result for right-to-manage wage bargaining.



unions, delegation becomes profit-oriented and the Cournot outputs move in the direction of the collusive output.

**Unilateral delegation by the union.** Now assume that only the unions delegate. Set  $\beta_1 = \beta_2 = 1$  in Eq. (8). Again consider a symmetric solution to Eq. (8)  $\gamma_1^* = \gamma_2^* = \gamma^*$ .<sup>7</sup>

**Proposition 2 ( Unilateral delegation by unions )** Denote  $\gamma^*(\alpha = 1) = \gamma^S$  and  $\gamma^*(\alpha = 1) = \gamma^B$ . Then  $\gamma^S < 1 < \gamma^B$ , and  $\gamma^{*'}(\alpha) < 0$  for  $\alpha \in [0, 1]$ . Consequently, there exists a unique critical value of  $\alpha$ ,  $\tilde{\alpha} \in (0, 1)$ , such that at all  $\alpha \in (\tilde{\alpha}, 1]$  there is  $\gamma^* < 1$  signifying gross wage bill oriented delegation, and at all  $\alpha \in [0, \tilde{\alpha})$  we have  $\gamma^* > 1$  signifying net wage bill orientation. At  $\alpha = \tilde{\alpha}$ ,  $\gamma^* = 1$ .

The nature of delegation for the union is similar to that of shareholders. In this sense, Proposition 2 produces a dual of Szymanski (1994) result. Over-employment (or under-employment) is not exclusive to shareholders delegation. We also note that the union's incentives are similar to, but not an exact mirror image of, the shareholders' incentives. For example, when  $\alpha = 0$  unions' delegation is completely devoid of any strategic motive (see Eq. (8)). In contrast, when  $\alpha = 1$  the shareholder's strategic motive does not fully disappear; see Eq. (7).

A particular implication of symmetry between the unions' strategic interest and the shareholders' strategic interest is that their incentive terms would be exactly identical, when they have the maximum bargaining power. The reason is that when a party is most powerful and it is the only side delegating, its sole objective of delegation is strategic, i.e. to steal market or jobs from its rival by aggressively expanding its own.

To see this, substitute  $\alpha = 0$  and  $\gamma = 1$  in Eq. (7). This gives

$$\frac{\partial \pi_i}{\partial \beta_i} = (\beta_i - 1)\theta \frac{\partial l_i}{\partial \beta_i} + \frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial \beta_i} = 0.$$

Next, substitute  $\alpha = 1$  and  $\beta = 1$  in Eq. (8) and obtain

$$\frac{\partial u_i}{\partial \gamma_i} = (\gamma_i - 1)\theta \frac{\partial l_i}{\partial \gamma_i} + \frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial \gamma_i} = 0.$$

These two equations are exactly mirror images of each other with  $\gamma_i$  and  $\beta_i$  swapping their places. So the optimal  $\beta_i(\alpha = 0)$  must be equal to optimal  $\gamma_i(\alpha = 1)$ .

**Proposition 3 (Symmetric strategic incentives)** Suppose  $\alpha = 0$  and  $\gamma_i = 1$  ( $i = 1, 2$ ) are exogenously given, and  $\beta^S$  is the symmetric optimal incentive of the shareholders. Similarly, suppose  $\alpha = 1$  and  $\beta_i = 1$  ( $i = 1, 2$ ) are exogenously given, and  $\gamma^S$  is the symmetric optimal incentive of the unions. Then it must be that  $\beta^S = \gamma^S$ .

**Bilateral delegation.** We now turn to bilateral delegation and consider Eqs. (7) and (8)

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<sup>7</sup>Symmetry is not essential for the results stated in the proposition. But it helps to establish monotonicity and comparative statics. The same argument applies to unilateral delegation by the shareholders.

together. We assume that all the second order conditions hold including the Cournot stability condition and a symmetric solution  $(\beta, \gamma)$  exists.<sup>8</sup>  $\beta(\alpha)$  and  $\gamma(\alpha)$  are continuous.

One important question in this context is whether the joint incentive term  $\beta\gamma$  (rather than  $\beta$  and  $\gamma$  independently) will exceed or fall short of 1. If  $\beta\gamma > 1$ , we will have underproduction relative to the duopoly level, whereas  $\beta\gamma < 1$  will give overproduction. However, Eqs. (7) and (8) do not directly give us any indication about the magnitude of  $\beta\gamma$ . Nevertheless, it is apparent that both the union and the shareholders commonly recognize the strategic effects of their incentive schemes. Therefore, despite their conflicting bargaining interests they would be able to ‘implicitly coordinate’ their choices to have a mutually beneficial effect on their bargaining pie.

Eqs. (7) and (8) can be utilized to derive the following equation that reconstructs the total effect of joint incentives from the individual player’s incentive choice:

$$\beta_i \gamma_i \left[ (\beta_i \gamma_i - 1) \theta \underbrace{\frac{\partial l_i}{\partial \beta_i \gamma_i}}_{(< 0)} + \underbrace{\frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial \beta_i \gamma_i}}_{(< 0)} + \underbrace{\frac{w_i l_i}{\beta_i \gamma_i}}_{(> 0)} \right] = 0. \quad (9)$$

The first term (inside the bracket) is the effect of an increase in the joint incentive  $(\beta_i \gamma_i)$  on the organizational surplus  $[s_i(l_i, l_j) - \theta l_i]$  occurring through own employment  $l_i$ . The second term is the strategic effect on the organizational surplus occurring through the rival firm’s employment. The third term is the ‘incentive adjusted’ wage bill, which is also an ‘approximate’ sum of the direct effects of  $\beta_i$  and  $\gamma_i$  on profit and union utility.<sup>9</sup>

In sum, the first two terms are the indirect effects, occurring through  $l_i$  and  $l_j$  respectively, and the third term is the direct effect. It is also the case that the direct effect is positive; the strategic effect is negative due to  $\partial s_i / \partial l_j < 0$ . So the sign of the first term depends on the relative strengths of the second and the third term, i.e. the strategic and the direct effects. If the direct effect dominates the strategic effect, then the first term must be negative, which requires  $\beta_i \gamma_i$  to exceed 1 (because  $\frac{\partial l_i}{\partial \beta_i \gamma_i} < 0$ ). Intuitively, if the strategic effect is relatively weak, markets or job stealing will not be easy. The players then can benefit by focussing on bargaining, and therefore enlarge the bargaining pie by restraining their production below the duopoly level by setting  $\beta_i \gamma_i > 1$ . On the other hand, if the strategic effect is greater than the direct effect, the players will be more concerned about losing market to their rival. In that case, they will set  $\beta_i \gamma_i < 1$  and will expand their output beyond the standard duopoly level.

**Proposition 4 ( *Bilateral delegation and strategic underproduction* )** (i) Suppose  $(\beta_i, \gamma_i)$ ,  $i = 1, 2$ , is a Nash equilibrium. Then for sufficiently small  $\alpha$ , we have  $\beta_i < 1$  and  $\gamma_i > 1$ . Conversely, for sufficiently high  $\alpha$  we have  $\beta_i > 1$  and  $\gamma_i < 1$ .

<sup>8</sup>For economy reasons these details are omitted here.

<sup>9</sup>It can be seen that, if  $l_i$  and  $l_j$  are held unchanged, we have

$$\frac{\partial \pi_i}{\partial \beta_i} \beta_i = \alpha \frac{s_i}{\beta_i}, \quad \frac{\partial u_i}{\partial \gamma_i} \gamma_i = (1 - \alpha) \theta \gamma_i, \quad \text{and} \quad \frac{\partial \pi_i}{\partial \beta_i} \beta_i + \frac{\partial u_i}{\partial \gamma_i} \gamma_i = w_i l_i.$$

(ii) Given any  $\alpha \in [0, 1]$ ,  $\beta_i \gamma_i > 1$  if and only if

$$w_i l_i > \left| \frac{\partial s_i}{\partial l_j} \right| \frac{\partial l_j}{\partial \beta_i \gamma_i} \beta_i \gamma_i, \quad i \neq j. \quad (10)$$

It is worthwhile to note that for Proposition 4 we have not imposed symmetry; in fact, to separate out the strategic effect from the own employment effect it is essential to include the subscripts  $i$  and  $j$ .

Suppose in a symmetric incentive equilibrium strategic underproduction occurs in *both* firms. Consequently, the organizational surplus or the bargaining pie will be larger than the duopoly level, giving rise to the possibility that both parties can be better off. In that case, the findings of the strategic delegation models and bargaining models are either contradicted or modified (Fershtman and Judd, 1987; Jones, 1989b; Szymnaski, 1997). Further, if the bargaining pie gets bigger, can it reach its maximum size, which is otherwise achievable only under a cartel? We try to answer this question next.

## 2.2 Bargaining Pie

The question of how the size of the bargaining pie is affected can be broken into two parts. How does the bargaining pie of firm  $i$  change with the joint incentive  $\beta_i \gamma_i$ , and how does the bargaining pie of firm  $i$  change when the bargaining power of the union change? The first question requires holding firm  $j$ 's incentives unchanged. But the second question requires accounting for changes in both  $\beta_i \gamma_i$  and  $\beta_j \gamma_j$  in response to a change in the exogenous parameter  $\alpha$ . So the second question is essentially about the comparative statics on the equilibrium incentive.

Let us begin with the first question. Noting that the bargaining pie of firm  $i$  ( $P_i = s_i - \theta l_i$ ) depends on the incentive terms only indirectly via employment, we derive (with the help of Eq. (3))

$$\frac{\partial(s_i - \theta l_i)}{\partial \beta_i \gamma_i} = (\beta_i \gamma_i - 1) \theta \frac{\partial l_i}{\partial \beta_i \gamma_i} + \frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial \beta_i \gamma_i} < 0.$$

Now notice that these two terms appear in Eq. (9), and since the third term in (9) is strictly positive, the sum of the above two terms must be negative; otherwise Eq. (9) will not be satisfied, which has been derived from the shareholders' and the workers' respective payoff maximization. Then it can be said that a *unilateral* increase in the joint incentives of firm  $i$  will reduce its own bargaining pie. This is so, because in duopoly raising incentives unilaterally implies losing market to its rival.

But that is only a part of the story. When an exogenous factor commonly affects the bargaining environments of both firms, and both firms symmetrically reset the equilibrium incentives, the bargaining pie of a given firm will face opposite effects from its own joint incentives and the joint incentives of its rival. Our second question directly relates to this. To analyze the effect of an increase in  $\alpha$ , we make a regulatory assumption.

**Assumption 2**  $|\frac{\partial l_i}{\partial x_i}| > |\frac{\partial l_i}{\partial x_j}|$  for  $i \neq j$ , where  $x$  represents a generic variable affecting employment.

This assumption states that the impact of firm  $i$ 's action (represented by  $x_i$ ) on its own employment will be stronger than the impact of the rival firm's action. With the help of this assumption and assuming symmetry we derive the effect of  $\alpha$  on a given firm's bargaining pie.<sup>10</sup>

$$P'_i(\alpha) = \left[ (\beta\gamma - 1)\theta + \frac{\partial s_i}{\partial l_j} \right] \frac{\partial(\beta\gamma)}{\partial \alpha} \underbrace{\left[ \frac{\partial l_i}{\partial(\beta_i\gamma_i)} + \frac{\partial l_i}{\partial(\beta_j\gamma_j)} \right]}_{< 0 \text{ by Assumption 2}}. \quad (11)$$

In Eq. (11) the sign of the overall expression depends on the first two terms. The sign of  $\partial(\beta\gamma)/\partial\alpha$  is ambiguous; in any case, if it is not zero, then our attention should be devoted to the first term. In a symmetric equilibrium when the bargaining pie is maximum in both firms, we must have the first term zero (given  $\partial(\beta\gamma)/\partial\alpha \neq 0$ ). That is,

$$\beta\gamma = 1 - \frac{1}{\theta} \frac{\partial s_i}{\partial l_j} \quad \text{for } i, j = 1, 2, i \neq j. \quad (12)$$

The second order condition confirms that the bargaining pie will be maximum<sup>11</sup>. Since  $\frac{\partial s_i}{\partial l_j} < 0$ ,  $\beta\gamma$  must be greater than 1. The largest pie is nothing but the collusive pie.

We show in Appendix that there exists a unique solution Eq. (12). Let this value of  $\beta\gamma$  be denoted as  $k$ , which will also be same regardless of delegation being one-sided or two-sided.

**Payoffs:** A related question is how the firm's (and union's) payoff is related to its bargaining power. In the general case, the effect of the union bargaining power  $\alpha$  on firm profit is ambiguous, for similar reasons explained in Chatterjee and Saha (2013). There are usually two effects of an increase in  $\alpha$  on the firm's profit. First is the direct effect, which is negative. Then there are two countervailing effects, one occurring through the union's incentive term  $\gamma_i$  and the other

<sup>10</sup>We write

$$P'_i(\alpha) = \left( \frac{\partial s_i}{\partial l_i} - \theta \right) \left[ \frac{\partial l_i}{\partial(\beta_i\gamma_i)} \frac{\partial(\beta_i\gamma_i)}{\partial \alpha} + \frac{\partial l_i}{\partial(\beta_j\gamma_j)} \frac{\partial(\beta_j\gamma_j)}{\partial \alpha} \right] + \frac{\partial s_i}{\partial l_j} \left[ \frac{\partial l_j}{\partial(\beta_i\gamma_i)} \frac{\partial(\beta_i\gamma_i)}{\partial \alpha} + \frac{\partial l_j}{\partial(\beta_j\gamma_j)} \frac{\partial(\beta_j\gamma_j)}{\partial \alpha} \right].$$

Use symmetry  $\frac{\partial l_1}{\partial \beta_1 \gamma_1} = \frac{\partial l_2}{\partial \beta_2 \gamma_2}$  and  $\frac{\partial l_2}{\partial \beta_1 \gamma_1} = \frac{\partial l_1}{\partial \beta_2 \gamma_2}$ , and  $\beta_1 \gamma_1 = \beta_2 \gamma_2$  to derive (11).

<sup>11</sup>We evaluate the second order condition at the maximum. First note that

$$\frac{\partial^2 s_i}{\partial l_j \partial \beta_i \gamma_i} = \underbrace{\left\{ \frac{\partial s_i^2}{\partial l_j \partial l_i} + \frac{\partial s_i^2}{\partial l_j^2} \right\}}_{< 0} \underbrace{\left\{ \frac{\partial l_i}{\partial(\beta_i\gamma_i)} + \frac{\partial l_i}{\partial(\beta_j\gamma_j)} \right\}}_{< 0} > 0, \quad i \neq j, \quad i, j = 1, 2.$$

In the above, we have used the assumption of symmetry to write  $\frac{\partial(\beta_i\gamma_i)}{\partial \alpha} = \frac{\partial(\beta_j\gamma_j)}{\partial \alpha}$  and  $\frac{\partial l_i}{\partial(\beta_i\gamma_i)} = \frac{\partial l_j}{\partial(\beta_j\gamma_j)}$ . This allows us to derive

$$P''_i(\alpha) = \left[ \theta + \underbrace{\frac{\partial^2 s_i}{\partial l_j \partial \beta_i \gamma_i}}_{> 0} \right] \left( \frac{\partial(\beta_i\gamma_i)}{\partial \alpha} \right)^2 \underbrace{\left[ \frac{\partial l_i}{\partial(\beta_i\gamma_i)} + \frac{\partial l_i}{\partial(\beta_j\gamma_j)} \right]}_{< 0} < 0.$$

occurring through the rival shareholders' incentive term  $\beta_j$ . Both effects are positive. Hence, the overall effect is ambiguous in all scenarios of delegation. Similar ambiguity arises also with unions' utilities. Given this complexity, we now consider an example to get a clear picture.

Now we would like to investigate with an example at what  $\alpha$  we can arrive at the solution  $\beta\gamma = k$ . The example will also help us ascertain how the payoffs vary with  $\alpha$ .

### 3 LINEAR DEMAND AND CONSTANT RETURNS TECHNOLOGY

Suppose the two goods are homogenous and the demand curve is linear as  $p = a - q_1 - q_2$ ; production exhibits constant returns to scale and  $q_i = l_i$ . Assume that  $a < 6\theta$ . The second stage output and wage of a firm resulting from efficient bargaining are ( for  $i \neq j$ ,  $i, j = 1, 2$ )

$$l_i = \frac{a - 2\beta_i\gamma_i\theta + \beta_j\gamma_j\theta}{3}, \quad w_i = (1 - \alpha)\gamma_i\theta + \alpha \frac{[a + \theta(\beta_i\gamma_i + \beta_j\gamma_j)]}{3\beta_i}.$$

We may also note that  $p = (a + \theta(\beta_i\gamma_i + \beta_j\gamma_j))/3$  and  $\theta\gamma_i < w_i < p_i/\beta_i$ .

Now consider the firms' optimal incentives. Profit in firm  $i$  is given as,

$$\pi_i = \left[ \frac{[a + \theta(\beta_i\gamma_i + \beta_j\gamma_j)]}{3} \frac{(\beta_i - \alpha)}{\beta_i} - (1 - \alpha)\theta\gamma_i \right] \frac{(a - 2\beta_i\gamma_i\theta + \beta_j\gamma_j\theta)}{3}, \quad i \neq j.$$

Maximizing  $\pi_i$  we obtain the shareholders' (implicit) reaction function (for  $i \neq j$ )

$$\frac{\partial \pi_i}{\partial \beta_i} = \frac{1}{9\beta_i^2} \left[ 4\gamma_i\beta_i^2\theta^2 \left\{ \frac{6\gamma_i\theta - a - \beta_j\gamma_j\theta}{4\theta} - \beta_i\gamma_i \right\} + \alpha \left\{ (a + \beta_j\gamma_j\theta)^2 - 4\beta_i^2\gamma_i^2\theta^2 \right\} \right] = 0.$$

Similarly, the union's utility in firm  $i$  is

$$u_i = \frac{(a - 2\beta_i\gamma_i\theta + \beta_j\gamma_j\theta)}{3} \left[ \frac{\alpha}{3\beta_i} \{a + \theta\beta_i\gamma_i + \theta\gamma_j\beta_j\} - \theta[1 - (1 - \alpha)\gamma_i] \right].$$

The first order condition for maximization is

$$\frac{\partial u_i}{\partial \gamma_i} = \frac{1}{9} [(a + \beta_j\gamma_j\theta)(3 - 4\alpha) + 6\beta_i\theta - 4(3 - 2\alpha)\beta_i\theta\gamma_i] = 0.$$

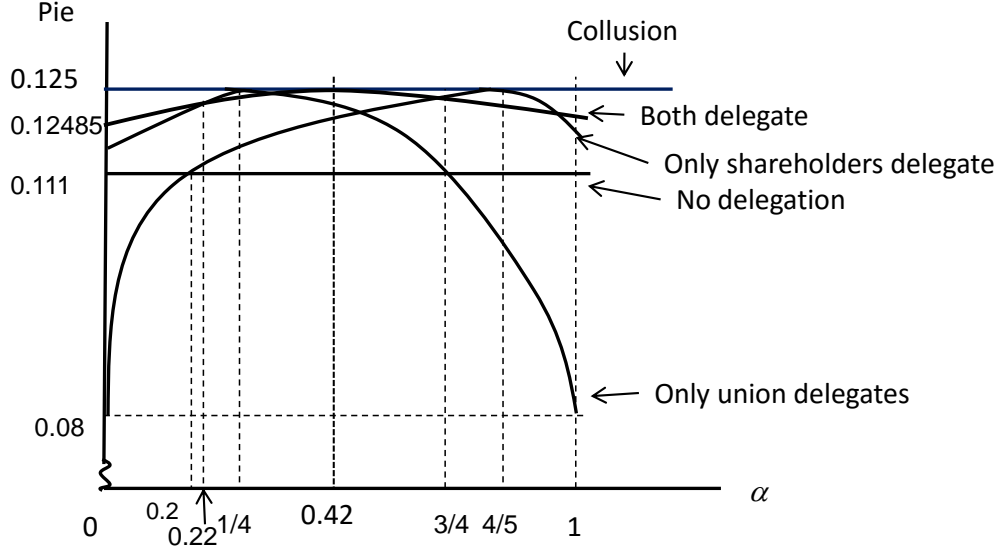
We assume there is a symmetric solution,  $\beta_i = \beta_j = \beta$  and  $\gamma_i = \gamma_j = \gamma$ . The symmetric solution simplifies the first order conditions as

$$\frac{\partial \pi}{\partial \beta} = \beta^2\gamma\theta [6\gamma\theta - a - 5\beta\gamma\theta] + \alpha [(a + \beta\gamma\theta)^2 - 4\beta^2\gamma^2\theta^2] = 0, \quad (13)$$

$$\frac{\partial u}{\partial \gamma} = (a - \beta\gamma\theta)(3 - 4\alpha) + 6\beta\theta(1 - \gamma) = 0. \quad (14)$$

Several observations can be made from the above two equations.

1. For outputs to be positive we must have  $a > \beta\gamma\theta$ . For restriction on  $\beta$  consider Eq. (13),



**Figure 1.** Bargaining pie for a firm in duopoly

in which  $[6\gamma\theta - \alpha - 5\beta\gamma\theta] \leq 0$  for  $\alpha \geq 0$ . Then we must have  $(6\gamma\theta - \alpha)/5 < \beta\gamma\theta$ . Since  $\beta\gamma\theta < \alpha$ , we also have  $6\gamma\theta - \alpha)/5 < \alpha$  and so  $\gamma\theta < \alpha$ . Similarly, consider Eq. (14) and rewrite it as

$$[3\alpha + 6\beta\theta - 9\beta\gamma\theta] - 4\alpha(\alpha - \beta\gamma\theta) = 0.$$

We must have  $[3\alpha + 6\beta\theta - 9\beta\gamma\theta] \geq 0$  for  $\alpha \geq 0$ . Then it follows that  $\beta\gamma\theta \leq (\alpha + 2\beta\theta)/3 = p$ , which must be less than  $\alpha$ . By setting  $p < \alpha$  we get  $\beta\theta < \alpha$ . In sum, we have  $\beta\gamma\theta < \alpha$ ,  $\beta\theta < \alpha$  and  $\gamma\theta < \alpha$ .

2. Set  $\alpha = 0$  in Eqs. (13) and (14) and obtain

$$\begin{aligned} 5\gamma\theta(1 - \beta) - (\alpha - \gamma\theta) &= 0 \\ (\alpha - \gamma\theta) - 3\beta\theta(\gamma - 1) &= 0. \end{aligned}$$

It follows that  $\gamma > 1$  and  $\beta < 1$ .

3. Set  $\alpha = 1$  in Eqs. (13) and (14) and write

$$\begin{aligned} \beta\gamma\theta[5\beta\gamma\theta + \alpha](1 - \beta) + (\alpha - \beta\gamma\theta)(\alpha + 2\beta\gamma\theta) &= 0 \\ 6\beta\theta(1 - \gamma) - (\alpha - \beta\gamma\theta) &= 0. \end{aligned}$$

From the above it is clear that  $\beta > 1$  and  $\gamma < 1$ .

4. Consider unilateral delegations and two extreme cases of bargaining – only the shareholders delegating with  $\alpha = 0$  and only the unions delegating with  $\alpha = 1$ . From the

shareholders' delegation we get  $\beta = (6\theta - \alpha)/5\theta$  and from the union's delegation we get symmetrically  $\gamma = (6\theta - \alpha)/5\theta$ . For these  $\gamma$  and  $\beta$  to be positive we must have  $\alpha < 6\theta$ .

5. It is also evident from Eq. (14) that regardless of  $\beta$  the union's best response is  $\gamma = 1$  when  $\alpha = 3/4$ . This .
6. Suppose only the shareholders delegate; that is set  $\gamma = 1$  in Eq. (13). We see that at  $\alpha = \frac{(\alpha-\theta)\theta}{(\alpha+\theta)^2-4\theta^2}$  the shareholders' best response is  $\beta = 1$ . Since  $\beta'(\alpha) > 0$  under unilateral delegation, we can assert that at  $\alpha > (<) \frac{(\alpha-\theta)\theta}{(\alpha+\theta)^2-4\theta^2}$  optimal  $\beta > (<) 1$ .
7. From Eq. (14) write the union's reaction function as

$$\gamma = \frac{\alpha(3-4\alpha)}{(9-4\alpha)\beta\theta} + \frac{6}{9-4\alpha}.$$

It is clear that at all  $\alpha > (<) 3/4$ ,  $\frac{\partial \gamma}{\partial \beta} > (<) 0$ . That is, above  $\alpha = 3/4$ , the union perceives  $\beta$  as strategic complement to  $\gamma$ , and below  $\alpha = 3/4$ , as strategic substitute for  $\gamma$ . That is to say, the strategic relationship between the two incentive terms is not uniform, when both the shareholders and the unions delegate. The shareholders' incentive  $\beta$  is also not always strategic substitute to  $\gamma$ ; however the analytical expression is not helpful enough to establish this clearly. When we present the simulation data, these issues may become somewhat clear.

Now we would like to see if the strategic underproduction condition (10) holds for the example we are considering. Substituting  $|\frac{\partial s_i}{\partial l_j}| = l_i$ ,  $\frac{\partial l_j}{\partial \beta_i \gamma_i} = \frac{\theta}{3}$  and the expression for  $w_i$  and then imposing symmetry we rewrite Eq. (10) as

$$\alpha \frac{\alpha}{\beta \gamma \theta} + 3 > \alpha + \beta. \quad (15)$$

It is not immediately obvious whether this condition will hold for any  $\alpha$ . Some careful scrutiny allows us to claim that if the joint incentive  $\beta\gamma$  is monotonic (either non-decreasing or non-increasing) then it will be always greater than 1, and strategic underproduction would occur in both firms. The following proposition states this formally.

**Proposition 5 (*Underproduction*)** *Suppose the demand curve is linear in two homogenous goods, and the production technology is constant returns to scale. Further, assume  $\beta\gamma(\alpha)$  is monotonic in  $\alpha$ . Then  $\beta\gamma > 1$  at all  $\alpha \in [0, 1]$ .*

Next, verify if the condition for bargaining pie maximization holds for our example. The pie maximization condition Eq. (12) reduces to  $\beta\gamma = 1 + \frac{\alpha}{\theta}$  (using  $q'(1) = 1$  and  $p'(q_i) = -1$ ). Since  $q = (\alpha - \beta\gamma)/3$ , we get  $\beta\gamma = (\alpha + 3\theta)/4\theta$ . Clearly, this is strictly less than  $\alpha$ , and hence within the feasible range of  $\beta\gamma$ .

Now it remains to be seen at what value of  $\alpha$ , we get  $\beta\gamma = (\alpha + 3\theta)/4\theta$ , and how profit and union's utility vary with the union's bargaining power. For this we resort to simulation.

### 3.1 Simulation

Table 1 shows the computations of the equilibrium incentives and outputs for  $\alpha = 2$  and  $\theta = 1$ . First we consider the no delegation case (Case 0). Bargaining power affects only the distribution of the bargaining pie, but leaves output and the (firm-level) pie unaffected. The aggregate duopoly pie for the industry is  $2 \times 0.111 = 0.222$ . The monopoly pie would be 0.25.

Next, Case 1 presents unilateral delegation by the shareholders. At  $\alpha = \frac{(a-\theta)\theta}{(a+\theta)^2-4\theta^2} = \frac{1}{9-4} = 0.2$  optimal  $\beta$  is exactly 1. Above  $\alpha = 0.2$ ,  $\beta$  is greater than 1 causing underemployment relative to Case 0, and below  $\alpha = 0.2$ ,  $\beta < 1$  causing over-employment. It is also noteworthy that profit rises above the no-delegation level only after  $\beta$  exceeds 1. That is, at  $\alpha < 0.2$  managers are oriented to sales maximization, and since both firms do the same, the outcome is overproduction and loss in profit. In other words, in this range of  $\alpha$  delegation suffers from a prisoner's dilemma problem. On the other hand, at  $\alpha > 0.2$  managers are oriented to profit maximization and underproduction helps to improve profit; delegation pays off.

Case 2 describes unilateral delegation by the union. Above  $\alpha = 0.75$ , the union leader is oriented to gross wage bill maximization, which leads to overproduction (relative to the no-delegation level) in both firms. Both unions are consequently worse off, a prisoner's dilemma phenomenon similar to the firm-side delegation (Case 1) at  $\alpha < 0.2$ . Below  $\alpha = 0.75$  unions orient their leaders to net wage bill maximization ( $\gamma > 1$ ), which in turn helps to reduce employment below the no-delegation level resulting in higher utility.

Finally, Case 3 concerns bilateral delegation. Here, at  $\alpha < 0.22$  the shareholders adopt sales orientation ( $\beta^* < 1$ ) and above  $\alpha = 0.22$  they adopt profit orientation. The unions continue to have gross wage bill orientation (as expected) above  $\alpha = 0.75$  and net wage bill orientation below  $\alpha = 0.75$ . The combined impact of these incentives on employment can be seen from the column for  $\beta\gamma$  and  $l$ .  $\beta\gamma$  remains greater than 1 and steadily decreases with  $\alpha$ ; consequently employment increases with  $\alpha$ .

Further, profit increases and union's utility decreases with an increase in  $\alpha$ . Profit under bilateral delegation is strictly increasing in  $\alpha$ ; in particular profit exceeds the no-delegation level at  $\alpha = 0.43$  (compare Case 0 with Case 3). On the other hand, union's utility from delegation is strictly decreasing in  $\alpha$ , but it remains above the duopoly level at all  $\alpha \leq 0.52$ . Thus, there is a range of  $\alpha$ , i.e.  $(0.43, 0.52)$ , over which both parties experience strictly positive gains from delegation.

Mutual benefits of (bilateral) delegation imply that the pie must have expanded. Indeed that is the case over the entire range of  $\alpha$ . This can be directly confirmed by comparing  $\pi + u$  of Case 3 with that of Case 0. Fig. 1 presents a visual illustration. The no-delegation pie is given by the flat line at 0.111, and the pie under bilateral delegation is inverted U-shaped. Not only is it strictly greater than 0.111, but it equals the collusive pie at  $\alpha = 0.42$ . Under unilateral delegation also the pie exceeds the no-delegation level, and it reaches the collusive level – at  $\alpha = 0.25$  when only the unions delegate and at  $\alpha = 4/5$  when only the shareholders delegate.



The pie maximization condition Eq. (12) reduces to  $\beta\gamma = 1 + \frac{q}{\theta}$  (using  $q'(\cdot) = 1$  and  $p'(\cdot) = -1$ ). Since  $q = (\alpha - \beta\gamma)/3$ , we get  $\beta\gamma = (\alpha + 3\theta)/4\theta$ .

Substituting  $\alpha = 2$  and  $\theta = 1$  we get the pie-maximizing  $\beta\gamma$  or  $k$  as 1.25. From Table 1 we can verify that  $\beta\gamma = 1.25$  at  $\alpha = 0.42$  (Case 3) and the pie is half of the monopoly (no-delegation) pie. For the other two cases of delegation as well we can identify  $\alpha$  corresponding to  $\beta$  or  $\gamma$  equal to 1.25.

Next, we comment on the strategic relationship between symmetric  $\beta$  and  $\gamma$ . As observed earlier, we indeed have asymmetric relationship. From the union's perspective  $\beta$  is strategic complement to  $\gamma$  at all  $\alpha > 0.75$ , and strategic substitute at all  $\alpha < 0.75$ . From the firm's perspective  $\gamma$  is strategic substitute to  $\beta$  at all  $\alpha > 0.2$  and strategic complement at all  $\alpha < 0.2$ . So only at  $\alpha \in (0.2, 0.75)$  they are strategic substitutes to each other. These can be read from Table 1 by tracking the movements in  $\beta$  and  $\gamma$  between unilateral delegation and bilateral delegation.

We summarize the key findings of this section with the following proposition.

**Proposition 6 (*Payoff-power inverse relationship*)** *Suppose the demand curve is linear in two homogenous goods, and the production technology is constant returns to scale. Then there exists a critical value of  $\alpha$  such that the bargaining pie is maximized in each firm and it is equal to the collusive pie. Furthermore, the firm's (union's) payoff is positively (negatively) related to the union's bargaining power  $\alpha$ .*

### 3.2 Decision to Delegate or not Delegate

We can extend the game backward to add a stage where both sides decide to delegate or not delegate by looking ahead at the symmetric equilibrium payoffs from the ensuing game. Potentially this would be a larger game involving four players. To simplify this we assume that shareholders of both firms come together and coordinate only their delegation decision, but their subsequent interactions will be fully non-cooperative as modeled in the paper. Similarly, unions of the two firms coordinate their delegation decisions, though subsequently they will act independently. This is a sort of semi-collusive game<sup>12</sup> and it can be suggested as follows.

- Stage 1: Suppose two firms jointly decide whether to delegate or not – a ‘yes-no’ decision; simultaneously the unions jointly decide whether to delegate or not. That is, both firms and both unions share the same delegation decision.

- Stage 2: Firms part company and so do the unions. If firms had decided ‘yes’ in stage 1, they independently hire their managers. Likewise, if unions decided ‘yes’ in the first stage, they would independently hire their union leaders.

- Stage 3: Each firm and union independently set their incentive terms  $(\beta_1, \gamma_1)$  and  $(\beta_2, \gamma_2)$ . This is a four-way decision.

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<sup>12</sup>Semi-collusion has been studied in many contexts. For example, Fershtman and Gandal (1994) studied collusion in the output market preceded by competition in R&D stage.

- Stage 4: The manager and the union leader in each firm bargain over employment and wage. If, in stage 1, one side (or both sides) had decided not to delegate, then in stage 4, the shareholders or the union will directly participate in decision making.

This allows us to compress the extended game as a two-player game in the first stage, and we can restrict our attention to the union and the shareholders of one firm only. By modeling the delegation decision in this way, we can easily solve the game.

Ideally, we need to consider the whole game in a fully non-cooperative spirit where the decision to delegate is independently taken by each firm and each union in stage 1, looking ahead all possible sub-games. But solving this game would require analyzing a large number of asymmetric cases, a tenuous task, needless to say without any clear benefit. Even in this very large game, one equilibrium path will be the symmetric payoffs with symmetric delegation decisions that we are focusing on in our smaller semi-collusive game.

Tables 2a-2c report the payoff matrices at three values of  $\alpha$ , 0.8, 0.5 and 0.1 respectively. It turns out that at all  $\alpha > 0.75$  the shareholders find delegation a dominant strategy while the unions find no-delegation as their dominant strategy. Thus, we have an outcome of asymmetric delegation with only the weaker party delegating. The outcome is reversed when  $\alpha < 0.22$ ; here, the shareholders being very strong abstain from delegation. At  $\alpha \in (0.22, 0.75)$  both parties delegate. Fig. 2 depicts the equilibrium delegation decisions. At  $\alpha = 0.75$  (alternatively 0.22) the union (alternatively shareholders) is indifferent between delegation and no-delegation.

Thus, we see an interesting scenario. When the players are most powerful on the bargaining front, they do not resort to delegation, their within-firm rival tries to cover their weakness in negotiation by resorting to strong bargaining oriented delegation. Production is consequently restrained below the duopoly level. This contradicts the premise of the majority of the strategic delegation models, where shareholders (assuming the most bargaining power against the workers) unilaterally delegate inducing overproduction. But we see that if their decision to delegate is made endogenous (as seen for  $\alpha < 0.22$ ), they would not have delegated at all. So the scenario commonly assumed in the managerial incentive literature, may not arise in a larger game.

## 4 CONCLUSION

In this paper we have developed a model of bilateral delegation in wage and employment bargaining in duopoly. Shareholders in each firm appoint a manager to negotiate on their behalf, and so do the workers by appointing a union leader. We first characterize and then compute (for an example) the optimal equilibrium incentive schemes. The combined impact of the shareholders' incentives and the workers' incentives is underemployment restraining production below the Cournot-Nash duopoly level. This acts like implicit collusion, as the bargaining pie expands (from the no-delegation level), and opportunities for mutual gains are created. But each player's payoff will be inversely related to his bargaining power. It may not be unfair to say that strong unions may facilitate firm collusion as much as strong firms facilitate union cooperation. In

Table 1: **Duopoly simulations**

$a = 2; \theta = 1$							
Case 0: No Delegation ( $\beta = 1; \gamma = 1$ )							
$\alpha$				l	u	$\pi$	$\pi + u$
1				0.333	0.111	0	0.111
0.75				0.333	0.083	0.028	0.111
0.5				0.333	0.056	0.056	0.111
0.42				0.333	0.056	0.056	0.111
0.25				0.333	0.028	0.083	0.111
0.20				0.333	0.222	0.89	0.111
0				0.333	0	0.111	0.111
Case 1: Only shareholders delegates ( $\gamma = 1$ )							
$\alpha$	$\beta$			l	u	$\pi$	$\pi + u$
1	1.29			0.235	0.0427	0.0819	0.12455
0.8	1.25			0.251	0.0406	0.0844	0.12500
0.75	1.23			0.256	0.0399	0.0851	0.12493
0.5	1.15			0.283	0.0349	0.0879	0.12278
0.25	1.03			0.323	0.0252	0.0892	0.11441
0.22	1.01			0.329	0.0234	0.0890	0.11241
0.2	1			0.333	0.0222	0.0889	0.11111
0.1	0.92			0.360	0.0140	0.0869	0.10097
0	0.80			0.4	0	0.08	0.08
Case 2: Only union delegates ( $\beta = 1$ )							
$\alpha$	$\gamma$			l	u	$\pi$	$\pi + u$
1	0.8			0.4	0.08	0	0.08
0.8	0.97			0.345	0.0832	0.0238	0.1070
0.75	1			0.333	0.0833	0.0278	0.1111
0.5	1.14			0.286	0.0816	0.0408	0.1224
0.25	1.25			0.250	0.0781	0.0469	0.1250
0.22	1.26			0.246	0.0776	0.0473	0.1249
0.2	1.27			0.244	0.0773	0.0476	0.1249
0.1	1.30			0.233	0.0757	0.0487	0.1244
0	1.33			0.222	0.0741	0.0494	0.1235
Case 3: Both delegate							
$\alpha$	$\beta$	$\gamma$	$\beta\gamma$	l	u	$\pi$	$\pi + u$
1	1.35	0.904	1.220	0.2599	0.0248	0.100	0.12480
0.8	1.26	0.980	1.230	0.2568	0.037	0.088	0.12491
0.75	1.23	1.0	1.232	0.2559	0.0399	0.085	0.12493
0.5	1.12	1.112	1.245	0.2516	0.057	0.068	0.12500
0.42	1.08	1.12	1.25	0.2501	0.062	0.063	0.12500
0.25	1.01	1.244	1.260	0.2467	0.075	0.050	0.12498
0.22	1	1.261	1.261	0.2463	0.078	0.047	0.12497
0.2	0.99	1.272	1.263	0.2457	0.079	0.046	0.12496
0.1	0.95	1.332	1.269	0.2436	0.087	0.038	0.12492
0	0.91	1.396	1.276	0.2414	0.096	0.029	0.12485

**Table 2a. Payoff matrix for  $\alpha=0.8$**

	Shareholders		
		Not delegate	Delegate
Union	Not delegate	0.0888, 0.0222	0.0406, 0.0844
	Delegate	0.0832, 0.0237	0.0367, 0.0881

Outcome: Union does not delegate, Shareholders delegate.

**Table 2b. Payoff matrix for  $\alpha=0.5$**

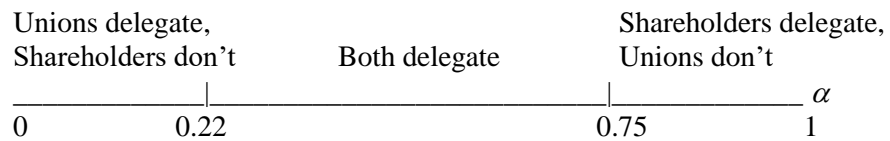
	Shareholders		
		Not delegate	Delegate
Union	Not delegate	0.056, 0.056	0.0349, 0.0879
	Delegate	0.0816, 0.0408	0.057, 0.068

Outcome: Both parties delegate.

**Table 2c. Payoff matrix for  $\alpha=0.1$**

	Shareholders		
		Not delegate	Delegate
Union	Not delegate	0.0111, 0.10	0.0140, 0.0869
	Delegate	0.0757, 0.0487	0.0872, 0.0376

Outcome: Union delegates, Shareholders don't.



**Figure 2. Decision to delegate**

either case, the industry outcome is nearly a collusive one. Because of delegation intra-firm conflicts help inter-firm cooperation.

Our analysis has abstracted from capital. It is not unreasonable to expect that capital will remain outside the union's negotiation. It remains to be seen whether that will help or hinder the shareholders. There are other issues such as the union's objective function. What if the union attaches different weight on employment than on wage? Some asymmetric cases might also be interesting - for example absence of union in one firm depicting a contrast between union and non-union sector. We believe that insights developed in this paper will go a long way in exploring such related issues.

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## APPENDIX

### Proof of Proposition 1

By definition  $\beta^*$  is the symmetric subgame perfect equilibrium incentive scheme, which is obvious from the discussion preceding the statement of the proposition. To see that  $\beta^S < \beta^*$  consider (7). Set  $\alpha = 0$  and solve for  $\beta$ ; this gives  $\beta^S$ . Clearly as  $\frac{\partial s_i}{\partial l_j} < 0$ ,  $\frac{\partial l_j}{\partial \beta_i} > 0$  and  $\frac{\partial l_i}{\partial \beta_i} < 0$ , we must have  $\beta^S < 1$ . Now plug back  $\beta^S$  in (7), but revert back to  $\alpha > 0$ . The first (bracketed) term is clearly zero at  $\beta^S$ , but the second (bracketed) term is strictly positive. Hence, for equation (7) to hold,  $\beta$  must be increased above  $\beta^S$ . Since by definition  $\beta^*$  satisfies (7),  $\beta^* > \beta^S$ .

Now for the upper bound on  $\beta^*$  set  $\alpha = 1$  in Eq. (7), which allows for the smallest strategic effect. Solve for  $\beta$ . This gives  $\beta^B$ . Since, for  $\pi = s_i(\cdot)(1 - \frac{\alpha}{\beta_i} - (1 - \alpha)\theta\gamma_i l_i)$  to be strictly positive (for delegation to be profitable) at all  $\alpha$  we must have  $\beta > 1$  when  $\alpha = 1$ . Hence,  $\beta^B$  must exceed 1. Now reconsider (7) and substitute  $\beta = \beta^B$  at any arbitrary  $\alpha < 1$ . The second term is now bigger in magnitude. Hence for equality to hold, the first term must remain positive and rise in magnitude, which requires reducing the negative part of the first term, i.e. reducing  $\beta$  below  $\beta^B$  (because  $\frac{\partial s_i}{\partial l_i} < 0$ ). Thus, we must have  $\beta^* < \beta^B$ .

Finally, for the comparative statics with respect to  $\alpha$  one derives from the first order condition of firm  $i = 1, 2$ ,

$$\begin{aligned} \frac{\partial^2 \pi_i}{\partial \beta_i^2} \beta_i^{*'}(\alpha) + \frac{\partial^2 \pi_i}{\partial \beta_i \partial \beta_j} \beta_j^{*'}(\alpha) + \frac{\partial^2 \pi_i}{\partial \beta_i \partial \alpha} &= 0, \\ \frac{\partial^2 \pi_j}{\partial \beta_j \partial \beta_i} \beta_i^{*'}(\alpha) + \frac{\partial^2 \pi_j}{\partial \beta_j^2} \beta_j^{*'}(\alpha) + \frac{\partial^2 \pi_j}{\partial \beta_j \partial \alpha} &= 0. \end{aligned} \tag{16}$$

From (7) derive  $\frac{\partial^2 \pi_i}{\partial \beta_i \partial \alpha} = \left[ s_i - \frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial \beta_i} \beta_i \right] > 0$  for  $i, j, i \neq j$ .

By the second order condition for profit maximization  $\frac{\partial^2 \pi_i}{\partial \beta_i^2} < 0$  for both  $i, j$ . By the Cournot stability condition  $\frac{\partial^2 \pi_i}{\partial \beta_i^2} \frac{\partial^2 \pi_j}{\partial \beta_j^2} > \frac{\partial^2 \pi_i}{\partial \beta_i \partial \beta_j} \frac{\partial^2 \pi_j}{\partial \beta_j \partial \beta_i}$ . By symmetry  $\frac{\partial^2 \pi_i}{\partial \beta_i^2} = \frac{\partial^2 \pi_j}{\partial \beta_j^2}$  and  $\frac{\partial^2 \pi_i}{\partial \beta_i \partial \beta_j} = \frac{\partial^2 \pi_j}{\partial \beta_j \partial \beta_i}$ . Therefore, the stability condition implies  $|\frac{\partial^2 \pi_i}{\partial \beta_i^2}| > |\frac{\partial^2 \pi_i}{\partial \beta_i \partial \beta_j}|$ , which in turn yields  $\frac{\partial^2 \pi_i}{\partial \beta_i^2} + \frac{\partial^2 \pi_i}{\partial \beta_i \partial \beta_j} < 0$ .

Now consider (16). By symmetry  $\beta_i^*(\alpha) = \beta_j^*(\alpha) = \beta^*(\alpha)$ , which can be obtained from (16) as

$$\beta^*(\alpha) = -\frac{\frac{\partial^2 \pi_i}{\partial \beta_i \partial \alpha}}{\frac{\partial^2 \pi_i}{\partial \beta_i^2} + \frac{\partial^2 \pi_i}{\partial \beta_i \partial \beta_j}} = -\frac{\left[ \frac{s_i}{\beta_i^2} - \frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial \beta_i} \frac{1}{\beta_i} \right]}{\frac{\partial^2 \pi_i}{\partial \beta_i^2} + \frac{\partial^2 \pi_i}{\partial \beta_i \partial \beta_j}} > 0.$$

At  $\alpha = 0$ ,  $\beta^* = \beta^S < 1$ . At  $\alpha = 1$ ,  $\beta^* = \beta^B > 1$ . Therefore, as  $\beta^*$  is continuous and increasing in  $\alpha$ , there exists a unique  $\alpha$ , namely  $\hat{\alpha}$ , at which  $\beta^* = 1$ . At all  $\alpha > \hat{\alpha}$  it is obvious that  $\beta^* > 1$  and at all  $\alpha < \hat{\alpha}$ ,  $\beta^* < 1$  due to the monotonicity of  $\beta^*$ .

### Proof of Proposition 2

The proof is analogous to the proof of Proposition 1. Fix  $\beta_i = 1$  in (8) for  $i = 1, 2$ . Then by setting  $\alpha = 0$  and  $\alpha = 1$  obtain  $\gamma^B > 1$  and  $\gamma^S < 1$  respectively. To show that  $\gamma^S \leq \gamma^* \leq \gamma^B$  apply the same reasoning as in the proof of Proposition 1. Next, by the continuity of  $\gamma$  there must exist a critical  $\alpha$  ( $\tilde{\alpha}$ ) such that  $\gamma^*(\tilde{\alpha}) = 0$ . That  $\tilde{\alpha}$  is unique follows from the fact that  $\partial \gamma^* / \partial \alpha < 0$ , which we establish below.

Using symmetry we derive from Eq. (8) for  $i = 1, 2$  and  $i \neq j$ ,

$$\gamma^*(\alpha) = -\frac{\frac{\partial^2 u_i}{\partial \gamma_i \partial \alpha}}{\frac{\partial^2 u_i}{\partial \gamma_i^2} + \frac{\partial^2 u_i}{\partial \gamma_i \partial \gamma_j}} = \frac{\left[ \theta l_i - \frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial \gamma_i} \frac{1}{\beta_i} \right]}{\frac{\partial^2 u_i}{\partial \gamma_i^2} + \frac{\partial^2 u_i}{\partial \gamma_i \partial \gamma_j}} < 0.$$

Since  $\partial \gamma^* / \partial \alpha < 0$  and  $\gamma^*(\alpha = 0) = \gamma^B > 1$  and  $\gamma^*(\alpha = 1) = \gamma^S < 1$ ,  $\tilde{\gamma}$  must be unique, and at all  $\gamma < (>) \tilde{\gamma}$ ,  $\gamma^* > (<) 1$ .

### Proof of Proposition 4

(i) Suppose  $((\beta_1, \gamma_1), (\beta_2, \gamma_2))$  is a Nash equilibrium and  $\beta_i$  and  $\gamma_i$  are continuous functions of  $\alpha$ ; hence they satisfy Eqs. (7) and (8) for  $i = 1, 2$ . Set  $\alpha = 0$  in (7) and (8). For (7) to hold we must have  $\beta_i < 1$  (since  $\partial l_i / \partial \beta_i < 0$ ) and for (8) to hold we must also have  $\gamma_i > 1$  since  $\partial l_i / \partial \gamma_i < 0$ . Then by continuity of  $\beta_i$  and  $\gamma_i$ , at all sufficiently small  $\alpha$  we will also have  $\beta_i < 1$  and  $\gamma_i > 1$ .

Next, set  $\alpha = 1$  in Eq. (8) from which we get  $\gamma_i < 1$ . From the expression of profit it becomes clear that  $\pi_i = s_i(\cdot)(1 - \frac{1}{\beta_i}) > 0$  only if  $\beta_i > 1$ . Again by continuity the same result will hold at sufficiently high values of  $\alpha$  close to 1.

(ii) Recall  $\partial \pi_i / \partial \beta_i = 0$  from Eq. (7)

$$(\beta_i - 1)\gamma_i \theta \frac{\partial l_i}{\partial \beta_i} + \left(1 - \frac{\alpha}{\beta_i}\right) \frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial \beta_i} + \alpha \frac{s_i}{\beta_i^2} = 0.$$

Substitute  $\frac{\partial l_j}{\partial \beta_i} = \frac{\partial l_j}{\partial (\beta_i \gamma_i)} \gamma_i$  for any  $i, j$ , and multiply both sides by  $\beta_i$  we get

$$(\beta_i - 1) \gamma_i \theta \frac{\partial l_i}{\partial \beta_i \gamma_i} \beta_i \gamma_i + \left(1 - \frac{\alpha}{\beta_i}\right) \frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial \beta_i \gamma_i} \beta_i \gamma_i + \alpha \frac{s_i}{\beta_i} = 0. \quad (17)$$

Similarly, consider  $\partial \pi_i / \partial \gamma_i = 0$  from Eq. (8)

$$\theta(\gamma_i - 1) \frac{\partial l_i}{\partial \gamma_i} + \frac{\alpha}{\beta_i} \frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial \gamma_i} + (1 - \alpha) \theta l_i = 0.$$

Substitute  $\frac{\partial l_j}{\partial \gamma_i} = \frac{\partial l_j}{\partial \beta_i \gamma_i} \beta_i$  (for any  $i$  and  $j$ ) and multiply both sides by  $\gamma_i$  to get

$$(\gamma_i - 1) \theta \frac{\partial l_i}{\partial (\beta_i \gamma_i)} \beta_i \gamma_i + \frac{\alpha}{\beta_i} \frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial \beta_i \gamma_i} \beta_i \gamma_i + (1 - \alpha) \theta \gamma_i l_i = 0. \quad (18)$$

Adding Eq. (17) to Eq. (18) we get

$$\begin{aligned} (\beta_i \gamma_i - 1) \theta \frac{\partial l_i}{\partial (\beta_i \gamma_i)} \beta_i \gamma_i + \frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial \beta_i \gamma_i} \beta_i \gamma_i + \alpha \frac{s_i}{\beta_i} + (1 - \alpha) \theta \gamma_i l_i &= 0 \\ \beta_i \gamma_i \left[ (\beta_i \gamma_i - 1) \theta \frac{\partial l_i}{\partial (\beta_i \gamma_i)} + \frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial \beta_i \gamma_i} + \frac{\alpha l_i}{\beta_i \gamma_i} \right] &= 0 \end{aligned}$$

which is precisely our Eq. (9). It is obvious from Eq. (9) that if condition (10) holds, then it must be the case that  $\beta_i \gamma_i > 1$ . Also, if  $\beta_i \gamma_i > 1$ , condition (10) must hold.

### Proof of the existence of the maximum pie

Consider Eq. (12). We would like to show that there exists a unique solution to this equation. Let us write  $\psi \equiv 1 - \frac{\partial s_i}{\partial l_j} \frac{1}{\theta}$  which is a function of  $\beta \gamma$  (rather than  $\beta$  and  $\gamma$  separately) via  $l_i(\cdot)$ . We are looking for a solution to the equation  $\beta \gamma = \psi(\beta \gamma)$ . Note that  $\partial s_i / \partial l_j$  is positively related to  $\beta \gamma$  (see Footnote 11). Hence,  $\psi'(\beta \gamma) = -\frac{1}{\theta} \frac{\partial^2 s_i}{\partial l_j \partial \beta \gamma} < 0$ . Now, as  $\beta \gamma \rightarrow 0$ ,  $\psi(\cdot) \rightarrow \infty$  (due to  $q_i \rightarrow \infty$ ), and as  $\beta \gamma \rightarrow \infty$ ,  $\psi(\cdot) \rightarrow 1$  (due to  $q_i \rightarrow 0$ ). On the other hand,  $\beta \gamma$  is a 45-degree line. Hence,  $\beta \gamma$  must cross  $\psi(\beta \gamma)$  exactly once. In other words, there is a unique solution of  $\beta \gamma$  to Eq. (12). Let this solution be denoted as  $k$ ; clearly,  $k > 1$ .

It is also clear that regardless of the two-sided or one-sided delegations, as the function  $\psi(\cdot)$  does not change except for its argument, the condition for the bargaining pie maximization is same. When only the shareholders delegate  $\beta$  must be equal to  $k$  (with  $\gamma = 1$ ), and when only the unions delegate we must have  $\gamma = k$  with  $\beta = 1$ .

### Proof of Proposition 5

First, let us see if the underproduction condition (15) holds when  $\alpha$  is zero (or close to zero). When  $\alpha = 0$ , the left hand side of condition (15) reduces to 3, and the right hand side will be strictly less than 1, because  $\beta(\alpha = 0) < 1$ . So if the right hand side has to exceed the left hand side, then  $\beta$  must exceed 3. This means  $\alpha/\theta > 3$  for that possibility to arise. Let us suppose

that is the case. Suppose at  $\alpha = 1$  the above inequality is reversed, which implies that  $\beta\gamma < 1$ ; condition (15) is both necessary and sufficient condition for  $\beta\gamma > 1$ . So the inequality at  $\alpha = 1$  becomes

$$\frac{\alpha}{\beta\gamma\theta} + 3 < 1 + \beta.$$

We know  $\beta < \alpha/\theta$  for any  $\alpha$ . So the above inequality implies

$$\frac{\alpha}{\beta\gamma\theta} + 3 < 1 + \beta < 1 + \frac{\alpha}{\theta}.$$

But this is not possible. If  $\beta\gamma < 1$ , we should have  $\frac{\alpha}{\theta} < \frac{\alpha}{\beta\gamma\theta}$ . Hence, this is a contradiction to our claim that  $\beta\gamma < 1$  at  $\alpha = 1$ .

What if the inequality (15) was reversed at some  $\alpha$ ,  $0 < \alpha < 1$ ? Suppose there exists an interval of  $\alpha$ ,  $0 < \alpha_0 < \alpha_1 < 1$ , such that at all  $\alpha \in (\alpha_0, \alpha_1)$  the inequality (15) is reversed implying  $\beta\gamma < 1$ . At all  $\alpha < \alpha_0$  and all  $\alpha > \alpha_1$  we have  $\beta\gamma > 1$ , with  $\beta\gamma(\alpha_0) = \beta\gamma(\alpha_1) = 1$ . But this is a contradiction to our assumption that  $\beta\gamma$  is monotonic in  $\alpha$ . Hence, we must have  $\beta\gamma > 1$  at all  $\alpha$ .

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